Can You Win The Seattle Mariners?

Major league baseball, in a brilliant marketing move, has decided to host a sweepstakes to win the Seattle Mariners. All that you have to do is collect the most newly minted contest baseball cards, each representing a famous player. You and $n-1$ other people enter the contest, and the MLB sends each of you a box containing between zero to M distinct cards. Each distinct card is in your box i.i.d. with probability q.

There is a catch, the MLB wants to give the Mariners to a group of people to manage, so each contestant is allowed to pool their cards together with $k-1$ people of their choosing. You desperately want to win, but don't know how many distinct cards you and $k-1$ other people need to win.

If the MLB decides to have $M = C \log_2 \binom{n}{k}$ cards, with $C \in (1,2)$, k chosen so that $\log(k)/\log(n) = \alpha$ for $\alpha \in (0,1)$ and q chosen to solve the equation $(1-q)^k = \rho$, with $\rho \in (0,1)$. Does there exist a triple of (α, C, ρ) where you can answer the following question:

What is the maximum proportion of the M distinct cards that a group of k people can get in this large n limit?