A study of natural predator’s effectiveness at controlling a tick population

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Introduction

Background

- In recent decades, tick-borne diseases have been spreading at an increasing rate in the United States.
- The geographic ranges of some tick populations have also increased, in part due to man-made changes to the environment.
    - Cases of Lyme Disease have tripled in since the 1990s.
    - The cattle industry has facilitated the spread of Gulf Coast ticks into regions of Oklahoma and Kansas.

These two factors have lead to new strategies to control tick populations.

Role of Host in Tick Dynamics

- Feeding on a host is essential for a tick to mature into the next stage of life or to reproduce.
- In some cases hosts may also be predators to the ticks.
    - For example, helmeted guinea fowl are hosts to larva and nymph deer ticks yet also predators to adult deer ticks.
- Hosts also play an essential role in the spread of tick-borne diseases.

Goal of Study

- The goal of this research is to study how hosts may impact tick abundance when at least one host is also a tick predator.
- We show that the tick population can be reduced by careful addition of these predator hosts. The overall impact of these hosts, however, may be more nuanced.

Initial Model

Base Model

We describe a hard-bodied tick population using a stage-structured model consisting of larva L, nymphs N and adults A:

$$\begin{aligned} L(t+1) &= \zeta_A \beta(A) s_a A(t) + (1-\zeta_L) s_L L(t) \\ N(t+1) &= \zeta_L s_l L(t) + (1-\zeta_N) S_N N(t) \\ A(t+1) &= \zeta_N s_n N(t) + (1-\zeta_A) s_A A(t) \end{aligned}$$

- ζ_i - probability a tick obtains a blood meal and matures
- s_i - probability that a tick in stage i survives for one time unit t
- $\beta(A)$ - adult fecundity

Allee Effect $\beta(A)$

Adult Fecundity is defined as:

$$\beta(A) = \hat{\beta}\frac{1 + aA}{1 + cA + aA^2}$$

If $a = 0$, then this term is a Beverton-Holt nonlinearity $\frac{\hat{\beta}}{1+cA}$ which describes negative density dependence.

If $a > c$, then this describes the component Allee effect, meaning there is a positive density dependence for some range of A.

Thus, we are assuming that an invading (i.e. small) tick population would benefit from increasing the population size as it increases the likelihood of finding a mate.

Basic Reproduction Number R_0

<u>DEF:</u> R_0 is the expected number of offspring produced by an individual throughout its lifetime in absence of density dependence.

$$R_0 = \frac{s_L s_N s_A \zeta_L \zeta_N \zeta_L \hat{\beta}}{(1 - s_L[1 - \zeta_L])(1 - s_N[1 - \zeta_N])(1 - s_N[1 - \zeta_N])}$$

Theorem

- If $R_0 < 1$, then the trivial equilibrium is locally asymptotically stable.
- If $R_0 > 1$, then the trivial equilibrium is unstable.

Existence of Equilibria

This model contains up to two positive equilibria:

$$\bar{A} = \frac{aR_0 - c \pm \sqrt{(c - aR_0)^2 - 4a(1 - R_0)}}{2a}$$

$$\bar{L} = \frac{s_A \zeta_A \beta(\bar{A})\bar{A}}{(1 - s_L[1 - \zeta_L])}$$

$$\bar{N} = \frac{s_L \zeta_L \bar{L}}{(1 - s_N[1 - \zeta_N])}$$

Theorem

- If $R_0 > 1$, then there exists one non-trivial equilibrium.
- If $R_0 < 1$ and $2ac + (1 + 2a - c)\sqrt{a(a - c)} > 1 + 2a^2$, then there are two non-trivial equilibria.
- If $R_0 < 1$ and $2ac + (1 + 2a - c)\sqrt{a(a - c)} < 1 + 2a^2$, then there is only the trivial equilibrium.

Stability

If we assume that all individuals mature in one unit of time t, we obtain the following stability condition.

Theorem

Suppose $\zeta_L = \zeta_N = \zeta_A = 1$.

- If one non-trivial equilibrium exists, then it is stable.
- If two non-trivial equilibria exists, then the smaller is unstable while the larger is stable.

Though we were unable to show it analytically, numerical simulations suggest that these stability results continue to hold for $0 < \zeta_i < 1$.

Incorporating Host Dynamics

Types of Hosts

DEF 1: A Predator Host is an animal which is a host to some developmental stages yet is a natural predator for others, such as:

- Helmeted Guinea Fowl
- Wild Turkeys
- Opossums

DEF 2: A Non-Predator Host is an animal which is a true host in the sense that it does not consume the ticks, such as:

- Hares
- Foxes

Host Assumptions

- Non-Predator Hosts (NPH), for immature stages, are smaller animals such a rats or squirrels which do not consume the larger adult ticks, and as such only effect the probability of a larva or nymph obtaining a blood meal.
- Non-Predator Hosts for adult ticks are larger mammals such as deer, which are hosts for the adult ticks and effect their probability of obtaining a blood meal.

Host Assumptions

- Predator Hosts (PH) are smaller animals such as guinea fowl or opossum which are hosts for immature ticks but also consume adult ticks. As such they affect the probability of immature ticks finding a host and reduce the probability of an adult tick reproducing.
- We assume hosts are not directly impacted by the tick's population.

PH Model

Modeling the interaction term between the ticks and their hosts as poissonian, we replace (1-ζ_i) with the probability of zero encounters to represent the proportion of a ticks which do not have an interaction with a host (blood meals and predation):

$$\begin{aligned}
L(t+1) &= s_a\beta(e^{-a_\gamma\gamma}(1-e^{-a_\Gamma\Gamma}A))e^{-a_\gamma\gamma}(1-e^{-a_\Gamma\Gamma})A(t)+s_Le^{-a_{\gamma_L}\gamma}L(t)\\
N(t+1) &= s_L(1-e^{-a_{\gamma_L}\gamma})L(t)+s_Ne^{-a_{\gamma_N}\gamma}N(t)\\
A(t+1) &= s_n(1-e^{-a_{\gamma_N}\gamma})N(t)+s_Ae^{-a_\Gamma\Gamma}e^{-a_\gamma\gamma}A(t)
\end{aligned}$$

- γ denotes PHs and Γ denotes adult NPHs.
- The average number of interactions are determined by proportionality constants a_i.

R_0 and Equilibria

$$R_0 = \frac{\hat{\beta} s_L s_N s_A (1 - e^{-a_{\gamma_L}\gamma})(1 - e^{-a_{\gamma_N}\gamma})(1 - e^{-a_\Gamma \Gamma}) e^{-a_\gamma \gamma}}{(1 - s_L e^{-a_{\gamma_L}\gamma})(1 - s_N e^{-a_{\gamma_N}\gamma})(1 - s_A e^{-a_\Gamma \Gamma} e^{-a_\gamma \gamma})}$$

Corollary

At most two positive equilibria may exist:

$$\bar{A} = \frac{aR_0 - c \pm \sqrt{R_0^2 a^2 + R_0(4a - 2ac) + (c^2 - 4a)}}{2ak}$$

$$\bar{L} = \frac{s_A \beta(kA) e^{-a_\gamma \gamma}(1 - e^{-a_\Gamma \Gamma})}{1 - s_L e^{-a_{\gamma_L}\gamma}} \bar{A}$$

$$\bar{N} = \frac{s_L(1 - e^{-a_{\gamma_L}\gamma})}{1 - s_N e^{-a_{\gamma_N}\gamma}} \bar{L}$$

The conditions for existence are the same as previously stated.

Effect of Predator Hosts

Theorem

When no Allee effect is present the tick population becomes extinct if,

$$\gamma > \frac{ln(\hat{\beta} s_L s_N s_A)}{a_\gamma}.$$

Theorem

Under an Allee effect the tick population becomes extinct if

$$\gamma > \frac{ln(2\sqrt{1+a+c}+c-2) - ln(a\hat{\beta} s_L s_N s_A)}{-a_\gamma}.$$

Note that both of these inequalities give an upper bound on the number of PHs that are needed to drive the tick population to extinction.

Tick Equilibrium against number of predator hosts

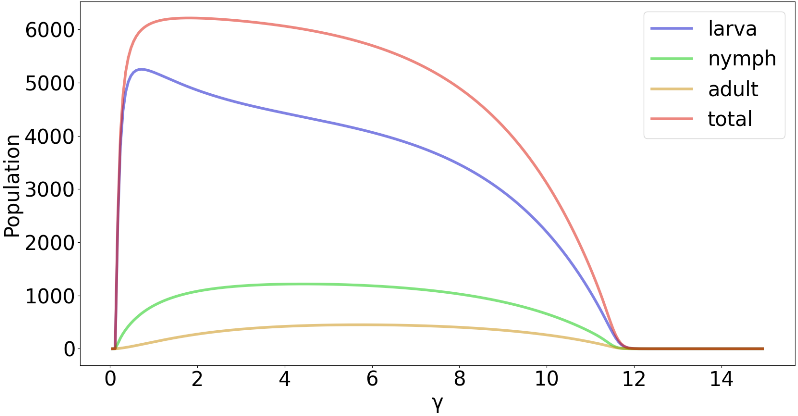


Figure 1: Without an Allee Effect

Tick Equilibrium against number of predator hosts

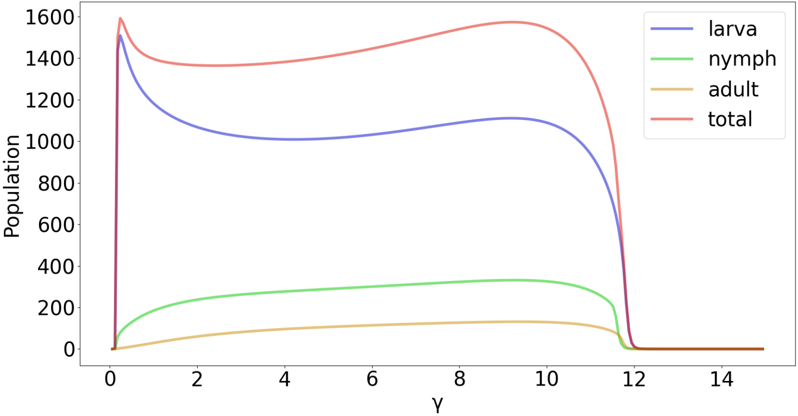


Figure 2: With an Allee Effect

Number of Local Population Maximums

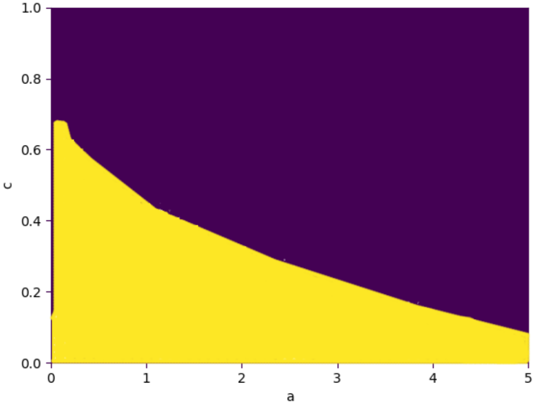


Figure 3: Yellow = 2, Purple = 1

NPH for Immature Stages

Next we introduce a NPH σ for the immature stages.

Since each tick stage only feeds once we let θ_i represent the proportion of immature ticks which are supported by the NPHs:

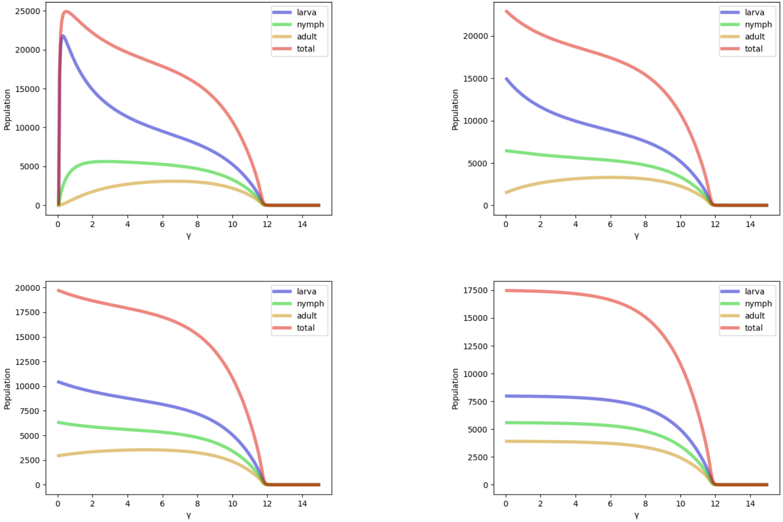
$$\begin{aligned}
L(t+1) &= s_a\beta(e^{-a_\gamma \gamma}(1-e^{-a_\Gamma \Gamma}A))e^{-a_\gamma \gamma}(1-e^{-a_\Gamma \Gamma})A(t) \\
&\quad + s_L(\theta_L e^{-a_{\sigma_L}\sigma} + (1-\theta_L)e^{-a_{\gamma_L}\gamma})L(t) \\
N(t+1) &= s_L[\theta_L(1-e^{-a_{\sigma_L}\sigma}) + (1-\theta_L)(1-e^{-a_{\gamma_L}\gamma})]L(t) \\
&\quad + s_N(\theta_N e^{-a_{\sigma_N}\sigma} + (1-\theta_N)e^{-a_{\gamma_N}\gamma})N(t) \\
A(t+1) &= s_n[\theta_N(1-e^{-a_{\sigma_N}\sigma}) + (1-\theta_N)(1-e^{-a_{\gamma_N}\gamma})]N(t) + s_A e^{-a_\Gamma \Gamma}A(t)
\end{aligned}$$

Non-Extinction Dynamics

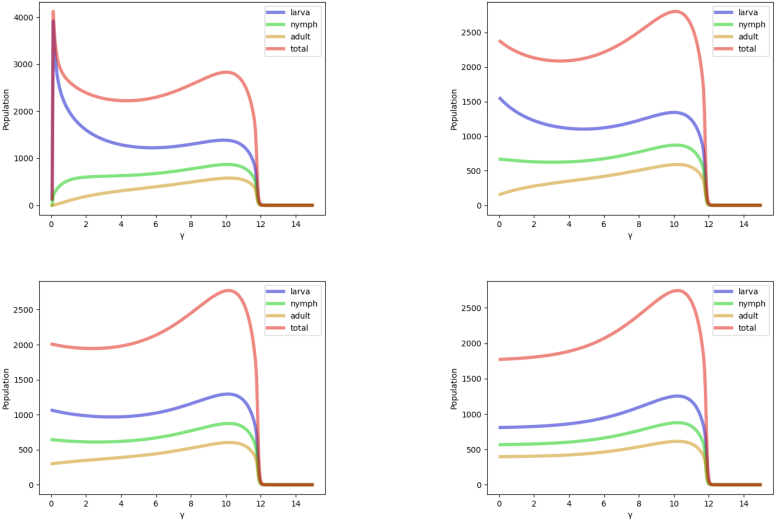
Corollary

The NPH model will result in an extinct population with the same parameters as Theorem 3 and 4.

- The graph will shift leftwards as σ increases, regardless of when extinction occurs compared to the previous model.
- This pulls the maximums of the tick population to the left, while keeping the extinction point about the same, increasing the interval in which the number of PHs is monotone decreasing to extinction.



(σ, θ) = (0,0), (σ, θ) = (20,0.33), (σ, θ) = (40,0.66), (σ, θ) = (60,1)



(σ, θ) = (0,0), (σ, θ) = (20,0.33), (σ, θ) = (40,0.66), (σ, θ) = (60,1)

Future Work

Seasonality

For deer ticks, the times in which they quest, that is actively seek blood meals, varies throughout the year and with location.

- Nymphs tend to quest earliest in the year starting in late winter.
- The larva follow in early spring climate, which varies based on location.
- Adult ticks tend to quest later in the fall.

It is important to understand how questing times of each life stage of the tick coincides with the foraging behavior of their given hosts.

Questing Curves Northeast

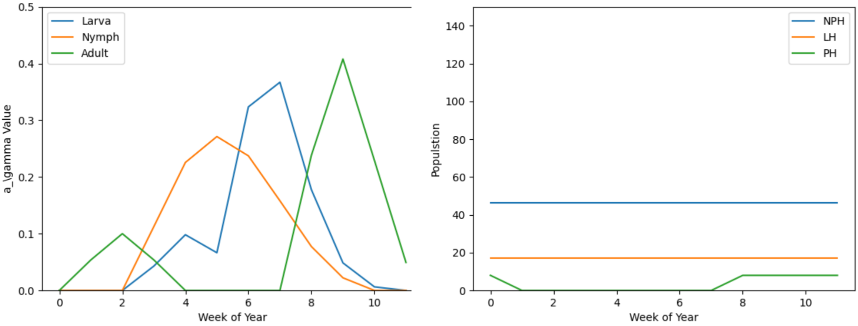


Figure 4: The questing curves of deer ticks & the foraging behaviors of Hares, deer and a variable PH

Initial Results

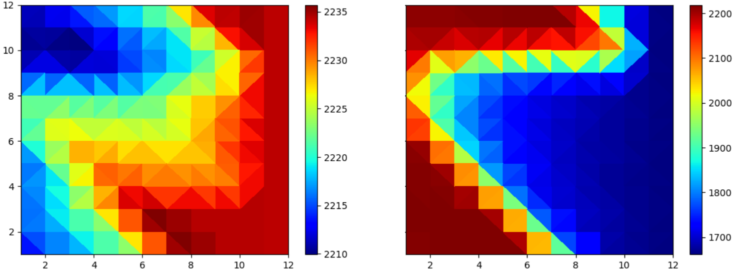


Figure 5: Yearly Mean of Total Population, γ = 10 (Left), 50 (Right).

- With the Horizontal Axis as the length that PHs are foraging for and the vertical axis being the time the Phs start foraging.
- We can see that the optimal foraging behavior for a PH changes depending on its population.

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